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On the determination of the vertical convective heat flux by observations of wind and temperature near the ground

by
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Summary

An attempt to measure the vertical convective flux of heat by simultaneous observation at a point of the vertical component of wind and the temperature has been continued, and fluctuations of frequency up to about 50 c/s have been recorded. The results obtained are about half of those of check determinations by the Bowen ratio method.

The evidence for the validity of the Bowen ratio method is reviewed, and considered satisfactory. The basis in theory of the "fluctuations" method is discussed, in particular the consequences of using instrumental means on various scales to replace the velocities and temperatures of fluid elements which figure in the theory. Observed similarities between results on different time scales are discussed. The possible effects of instrumental defects are examined. No firm reason for the discrepancy between the two methods of measuring the heat flux is found.

An appendix describes the apparatus used for recording at the highest frequencies.

I. Introduction

In M.R.P. 740 we reported an attempt to measure the vertical convective flux of heat near the ground by simultaneous observation of temperature and the vertical wind component, and gave reasons for our belief that the results we obtained were systematically low by a factor of about two. We have continued this type of observation, making certain changes in detail. We have now available about twice the amount of material, and we have investigated some of the possible sources of error, but consistently obtain similar results. In the present report we collect together all our observations and discuss known sources of error, mainly in the hope of obtaining advice on our difficulties.

The technique we have used is a popular one (see e.g. Hewson 1952), but few of its users appear to have subjected it to any serious test. Swinbank (1951, 1952) and his co-workers, and McCready (1953), form the only exceptions known to us, and we later compare their experience with our own.

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II. Apparatus and procedure

The apparatus was for the most part the same as that described in M.R.P. 740. The previous work had been carried out at a height of 25 cm, the response of the recording apparatus being controlled by suitably damped galvanometers of free period 1/10 sec. Most of the observations now reported were made at a height of about 150 cm with these galvanometers, the same site being used. In addition observations have been taken at a height of 150 cm with a similar apparatus incorporating galvanometers of period 1/60 sec. Details of this apparatus and its performance are given in Appendix I.

The method of dealing with the charts was unchanged. The records with the 1/60 sec. galvanometers were measured at intervals of 1/30 sec., each record being long enough to give 300 such readings. Only one set (5 records) of observations with this apparatus has been completely reduced.

III. Results

The results of all observations so far reduced, including those previously discussed in M.R.P. 740, are set out in Table I. The last three columns of this table give the heat flux as determined by the fluctuation technique and the heat balance - Bowen ratio method, and the ratio of the two determinations. The discrepancy previously found is repeated in the new results; the mean value of the ratio, weighted to take account of the number of records in each observation, is 0.44. The likely errors of the various results will be discussed in more detail in the following sections of this report, but the position may be summarised by saying that within the accuracy of the measurements the fluctuation technique as we have applied it gives a value for the heat flux which is half that obtained by the Bowen ratio method. In lapse conditions the magnitude of the ratio of the two determinations found at a height of 140 cm is rather greater than that at 25 cm, but this does not hold for the inversion case, observation 12. There is a very high degree of internal consistency in the results, compare for example observations 5, 6 and 13, and particularly 2, 8 and 12. These three inversion cases constitute, as nearly as is possible in work in the field, a repetition of the same experiment, and the results are almost identical (Observation 1, the only other inversion examined, was a very different case, with much higher wind speed).

In view of this consistency we suggest that we have minimized casual errors of observation and that the discrepancy disclosed in Table I must be due to an important systematic error in our methods of measurement, or in our application of the principles of either the Bowen ratio or fluctuation method, or in these principles themselves. We will examine these possibilities in turn.

IV. Discussion of the results

IV.1. The Bowen ratio method

This is a well known method for the separation of the heat exchanges at a surface due to convection of heat and evaporation, when the sum of these is known. Its applicability to a natural cropped surface has been questioned, e.g. by Pasquill (1949). Since we rely on it for a standard of comparison we will recapitulate the evidence for its validity.

It is not a method securely founded in theory since it implies that the transfer of heat and water vapour near the surface can be described by means of eddy diffusion coefficients, and in its practical application these coefficients are assumed equal. Rider and Robinson (1951) working on the site of our present experiments, made observations which could be explained in terms of eddy diffusion coefficients related by a constant but undetermined factor (possibly unity). Rider (1953) working at Cardington has extended this result and shown that this constant factor is normally near unity;

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though he reported a number of occasions (about 20%) when the validity of the method was in doubt. Rider's results concerning the ratios of the eddy diffusion coefficients are set out in Table II, where K_W, K_M, and K_H represent respectively the eddy diffusion coefficients for water vapour, momentum and heat. Table II also contains the standard deviations of these ratios estimated from the accuracy of the measurements of the various components as assessed by Rider. It will be seen that only observation No. 35 is exceptional and it has been omitted in the means. Observation No. 48 is doubtful, but has been included. The difference from unity of the observed ratios appears to be significant, but in the case of K_H/K_W with which the Bowen ratio method is concerned, this difference is small, and we have neglected it in the computation of Table I.

Equality of the eddy diffusion coefficients implies similarity of the profiles of wind, humidity, and temperature since, within the accuracy of the gradient measurements, the fluxes of heat and water are known to be constant over the range of heights concerned, and the assumption, which is usually made, that the flux of momentum is constant, leads to a consistent interpretation of observations. This similarity of profiles is always observed on the Kew site up to a height of 200 cm, within the accuracy of the measurements. Rider and Robinson (1951) gave examples; a further one, associated with observation No. 10 of the present series, is shown in Fig. 1. We consider that we are justified in applying the Bowen ratio method to measurements on the Kew site, and in particular that there is no reason in principle for the method systematically to overestimate the vertical convective heat flux at this site by a factor of about 2.

The accuracy of the measurements remains to be considered. The greatest source of random error is probably the measurement of heat flux into or from the soil layers, and we have minimised this by choosing occasions when, for a given radiation field, this term is small. We are satisfied that there is no systematic error in this term sufficient to affect our results. The most likely source of large systematic error is faulty measurement of one or more of the radiation terms, since the required convective heat flux, at least in the lapse conditions investigated, is often of the order of 10% of the incident solar energy, and 5% errors in radiation measurements are not uncommon. In the Bowen ratio method an error in the measurement of radiation is divided between the heat flux and evaporation, but if the evaporation is determined directly, as for example by Pasquill (1949) and Rider (1953), the whole error is included in the heat flux. In these circumstances a systematic error of 10% in solar radiation measurement can lead to an error of order 100% in the ratio K_H/K_W for lapse conditions. The radiometers used at Kew are frequently standardised, the solar radiation being referred to a substandard Ångström pyrheliometer known to be in accord with international standards (Eldridge 1952), and the terrestrial radiation to black body sources (Robinson 1947). The Gier and Dunkle type radiometer used in observation 13 (see Appendix I) had been calibrated against both types of substandard, with satisfactory consistency. We therefore do not believe that there is a large systematic error in the radiation measurements, and in consequence the Bowen ratio heat flux results in Table I are considered to be a satisfactory standard of comparison. We estimate that the casual error of these results does not exceed ± 2 mw cm^{-2} or $\pm$ 20%, whichever is the greater.

IV.2. Theory of the "fluctuations" method

Various ways of defining the heat flux in terms of the instantaneous winds and temperatures were mentioned in M.R.P. 740. Probably the most general is that set out, e.g. by Starr (1952), who considers all the forms of energy entering and leaving a specified domain in the atmosphere.

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His expression contains the terms

$$\int_A \int_t (\rho I + p) v_n \, dA \, dt \qquad - (1)$$

which may be written

$$\int_A \int_t \rho c_p T v_n \, dA \, dt \qquad - (2)$$

for an atmosphere of constant composition, where I is the specific internal energy, p the pressure, T the temperature, and v_n the normal component of velocity relative to a surface element dA at time t. If we apply this expression to a system defined by the earth's surface, a vertical wall, and a horizontal surface, and assume that the integral is zero over the wall (i.e. no horizontal advection of enthalpy) we have for the flow of enthalpy through the upper surface

$$\int_A \int_t \rho c_p T w \, dA \, dt \equiv A.t.c_p \overline{\rho w T} \qquad - (3).$$

The assumption underlying the present experiment is that the integration can be performed over a domain for which the flow of mass through the wall and the vertical mass flow are both zero i.e. $\overline{\rho w} = 0$ so that

$$F_H \equiv c_p \overline{\rho w T} = c_p \overline{(\rho w)' T'} \qquad - (4)$$

is now the vertical flux of heat, to be compared with the flux of heat into the domain at the earth's surface.

Three difficulties arise. Firstly we observe at a constant level, whereas the condition $\overline{\rho w} = 0$ defines a constant pressure surface. It is therefore implied that systematic vertical flux of mass through a constant level is not an essential part of the heat transfer mechanism. Secondly we measure w rather than ρw and deduce $\overline{w}$ and w' not $\overline{\rho w}$ and $(\rho w)'$ (see M.R.P. 740). Thirdly we observe only at one point, on the assumption that the time and space integrations are equivalent, as in the corresponding problem of statistical mechanics.

We shall not discuss the first difficulty further. The implication seems reasonable, though we know of no demonstration of its truth. The immediate consequence of the second difficulty is that we must approximate to $\overline{\rho w T}$. Writing $\rho = \bar{\rho} + \rho'$ etc. we find

$$\overline{\rho w T} = \bar{\rho} \bar{w} \bar{T} + (\bar{\rho} \, \overline{w'T'} + \bar{T} \, \overline{w'\rho'}) + \bar{w} \, \overline{\rho'T'} + \overline{\rho'w'T'} \qquad - (5)$$

or by making immediate use of $\overline{\rho w} = 0$

$$\overline{\rho w T} = \overline{\rho w T'} = \bar{\rho} \, \overline{w'T'} + \bar{w} \, \overline{\rho'T'} + \overline{\rho'w'T'} \qquad - (6).$$

We now neglect the pressure fluctuation p', the term $\overline{\rho'T'}$ so that $T'/\bar{T} = -\rho'/\bar{\rho}$, and the term $\overline{\rho'w'T'}$ so (5) becomes

$$F_H = c_p \overline{\rho w T} \simeq c_p \bar{\rho} \bar{w} \bar{T} \qquad - (7)$$

Similarly (6) becomes

$$F_H = c_p \overline{\rho w T'} \simeq c_p \bar{\rho} \, \overline{w'T'} \qquad - (8).$$

The neglect of p' and $\rho'T'$ may be justified numerically in cases (such as those we have investigated) where the fluctuations of wind velocity, and hence the Reynolds pressures, are not extremely large. The neglect of the triple correlation term implies no correlation between w' and $|T'|$ which seems reasonable, so that (7) and (8) would be expected to give F_H with an accuracy of a few parts per cent in the cases here considered.

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Of these expressions (7) does not depend on the condition $\overline{\rho w} = 0$, it is therefore valid as an expression for the flow of enthalpy past the instrument during the period of observation; (8) on the other hand contains $\overline{\rho w} = 0$ and should be applied only when this condition holds.

So far we have been concerned with expressions involving the properties of elements* of the fluid; ρ, w, T, in (7) and (8) are instantaneous values for such an element. Instrumental readings are necessarily means over a domain in space and time determined by the size and time constant of the instrument. If we denote such instrumental means by $[\rho]$, $[w]$ and $[T]$ we have from (7)

$$\overline{F_H} \simeq c_p \overline{[\rho][w][T]} \simeq c_p [\overline{\rho}][\overline{w}][\overline{T}] \phi \qquad - (9)$$

to the same degree of approximation as (7). We cannot, however, establish an expression analogous to (8), involving $[w]'$ and $[T]'$. We conclude from this examination, by comparing expressions (7) and (9), that a flux of enthalpy, or flux of heat for a constant mass system, determined in this way from instrumental readings, is independent of the scale of the instrument used. We conclude also that expression (8) is valid only for a constant mass system, and then only if the observations of w' and T' are on the scale of the fluid elements.

We turn now to the results of our observations. The ideal constant mass system is never realised, $[w]$ being found always to be numerically 10^2 to 10^3 times greater than that required to account for the heat flux. (The value of $[w]$ normally fluctuates about a large mean; if exceptionally a mean value of the required magnitude were to be present the individual values of $[w]$ are so large that tens of thousands of consecutive readings would be required to eliminate the statistical fluctuations of $[w]$. Although $\bar{\rho}\bar{w}$ is not measured, it is obviously not zero, and (9) therefore gives the flux of enthalpy past the instrument, which is not an interesting quantity. We are left with no theoretically sound method of estimating the heat flux from the observations. The values quoted in Table I are the result of computing $\overline{[w]}$, $\overline{[T]}$ and $\overline{[w][T]}$ and writing

$$F_H = c_p \bar{\rho} \left(\overline{[w][T]} - \overline{[w]}\,\overline{[T]}\right) = c_p \bar{\rho} \overline{[w]'[T]'} \qquad - (10)$$

which is an expression which we have failed to justify theoretically. (Indeed we have shown $c_p [\overline{\rho}][\overline{w}][\overline{T}]$ to be a good approximation to the total enthalpy flux).

This procedure, of observation and computation, is similar to that adopted by other workers (e.g. Swinbank and McCready whose results are discussed in Section V), although there are differences in interpretation of the observations. We can only conclude from the preceding analysis that there should be no significance in heat fluxes computed from (10) but we see from Table I that they bear a simple relation to the true values. There is a further point of interest. In M.R.P. 740 we made some attempt to determine the periods of the eddies responsible for the heat transport, with rather paradoxical results. We wrote there "The results of filtering out the short period components (Table III (of M.R.P. 740), 14th September, 1950 and 6th June, 1951) suggest indeed that the main effect is due to fluctuations of period less than 2 sec., which is hardly in accord with the results obtained on 2nd July, 1951 using 2 sec. galvanometers". Table I (of the present report) shows that comparable results are obtained on three very different

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* It is of course a fundamental assumption, here and in all dynamical treatment of atmospheric motion, that a scale of motion exists which is sufficiently large compared with the molecular scale, and at the same time sufficiently small compared with the smallest scale on which turbulent mixing occurs, for a volume of fluid to be defined and followed as an entity in its motion. This is the fluid element.

ϕ Here and in the sequel we conveniently ignore the fact that the instrument is not a linear detector so that instrumental means are not the means defined by the integral (2).

scales, and we recall in this connection that Swinbank's work has been done in the main with 2 sec. galvanometers, so that we can be satisfied that our only observation on this scale is not unrepresentative. The independence of scale of observation recalls the similar independence to be expected if the heat flux is determined by (9); again we find a regularity in the results which we cannot justify theoretically but which suggests that they are in fact at least related to the heat fluxes.

These questions can be seen to be related to the third major difficulty of the method which we mentioned above, in that they would not arise if the means could be taken over the postulated domain for which $\overline{\rho w} = 0$. This would involve space as well as time integration, and is in practice impossible. The customary assumption of equivalence of time integrals and space integrals is by no means obviously true in the present connection - it implies that a scale of observation exists on which the mean values of temperature and vertical wind at a point are not correlated. Some aspects of our results can indeed be explained by saying that we have computed only one contribution to the flux and that some form of patterned flow in space is an essential feature of the heat transfer mechanisms but we do not think our method of computation is sufficiently well founded to allow us to press this explanation. Indeed we conclude that the method we have adopted for computation of the heat flux, though the customary one and at first sight reasonable, has not a firm theoretical basis. We hesitate to attribute our anomalous results to this until all other possibilities have been explored, in view of the regularities we have found and cannot explain, and of the success claimed by other users of the method. We are unable to suggest a more satisfactory method for reduction of the results.

IV.3. Duration of the observations

There is no way, a priori, of fixing the length of time over which the observations are to be made. If the integration in expression (2) were possible in practice it would be necessary to carry it out over a time and area such that $\overline{\rho w} = 0$ held to a very close approximation. Experience, our own and others', suggests that this condition would not in general be attained in observations at one point.

In M.R.P. 740 we followed Swinbank in an analysis which we interpreted as showing that we had taken mean values over a sufficiently long period. We had a series of independent observations of vertical velocity w (we now drop the distinction between the instantaneous w and the instrument mean $[w]$), $w_1, w_2, \text{----} w_p$, and corresponding temperature observations $T_1, T_2, \text{----} T_p$. We formed the mean product of the fluctuations $1/p \sum_{q=1}^{p} w'_q T'_q$ proportional to the heat flux as determined from p observations, F_p. We then divided the series of observations into r consecutive sets each of n observations $(rn = p)$ and found for each set the means of w and T and the product of the fluctuations from these means, taking the mean of this over the r sets, i.e.

$$\frac{1}{p} \sum_{a=0}^{r-1} \left[\sum_{q=an+1}^{n(a+1)} w_q T_q - \frac{1}{n} \sum_{q=an+1}^{n(a+1)} w_q \sum_{q=an+1}^{n(a+1)} T_q \right]$$

which is proportional to the heat flux as determined from n observations, F_n. We did this for a series of values of n and formed the ratio F_n/F_p, exhibiting the results in Table III of M.R.P. 740. We argued that since F_n/F_p was effectively unity over a wide range of n/p, the p observations were sufficient to give the heat flux. We have repeated this analysis for a number of occasions, and find that the behaviour of the case studied in detail in M.R.P. 740 (14th September, 1950) is not typical. The behaviour of individual runs of 200 or 300 readings varies widely as may be seen from Table III, which shows the results for the 5 sets of 300 readings

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comprising observation No. 13 of Table I and the 5 sets of 200 readings comprising observation No. 10 of that Table. The mean values for these two observations are plotted in Fig. 2, in which we have also plotted the mean of the five cases similarly analysed by Swinbank in M.R.P. 429. (Swinbank's series were of 160 observations. He did not form the mean product for the full series, so we have fitted this result for $n = 55$ to the curve determined from our own work). This curve has some curious features which are discussed further in Appendix II; here we consider its implications in regard to the heat flux determinations.

Figure 2 incorporates observations of very different types. Swinbank's series were obtained with 2 sec. galvanometers and comprise 160 equally spaced readings over a duration of 5 minutes. Our series are of 200 readings at intervals of 1/4 sec. (1/10 sec. galvanometers) and 300 readings at intervals of 1/30 sec. (1/60 sec. galvanometers). Thus there is thirty fold range in total time of observation but only a twofold range of p, and we are not justified in extrapolating over a wider range of p. If however the extrapolation is made, the curve suggests that the heat flux is doubled by a tenfold increase of p, or increased by 10% if p is doubled. We cannot therefore be satisfied, by this type of argument, that we have taken sufficient independent consecutive readings of w and T.

IV.4. Instrumental errors

Certain instrumental errors were mentioned in M.R.P. 740, where it was suggested that they were small, though unfortunately most of them tend to reduce the computed heat flux. The main sources of error seem to be poor synchronisation of wind and temperature records, temperature coefficient of the anemometer, and failure to follow the variations in direction of the horizontal component of wind.

We have no further evidence concerning the effect of lack of synchronisation between the wind and temperature traces. This can occur to any serious extent only by distortion of the photographic paper. We have estimated the error as less than 5%, and some confirmation of this is obtained from the results with the high speed galvanometers (observation 13). We see from Fig. 2 that in this observation combination of the measured winds and temperatures in groups of 5 reduced the computed flux by only 10% in spite of the smaller number of independent observations. Combination of observations in this way greatly reduces the possibility of error due to lack of synchronisation.

In M.R.P. 740 we noted that the temperature coefficient of the anemometer was about 1.5% per degree C. This was a rough determination. Analysis of the circuit (by Mr. D.R. Samuel) shows that the coefficient should vary considerably with wind speed, but that 2% /°C is a reasonable value at the speeds encountered in the observations. We had previously checked by trial that a temperature coefficient of this magnitude would have little effect on the computed results for the heat flux. This is easily confirmed. It is assumed that the instrument is calibrated at the mean temperature of the run. If we denote instrumental determinations of vertical wind by W, true values by w, the temperature fluctuations by T', and the temperature coefficient by α we have

$$\begin{aligned}
\text{measured heat flux} &= \bar{\rho}\, c_p \left(\overline{WT} - \overline{W}\,\overline{T}\right) \\
&= \bar{\rho}\, c_p \left[\overline{wT(1+\alpha T')} - \overline{T}\left(\overline{w} + \alpha\overline{wT'}\right)\right] \\
&= \bar{\rho}\, c_p \left[\overline{wT} - \overline{w}\,\overline{T} + \overline{w(\overline{T}+T')\alpha T'} - \overline{T}\alpha\,\overline{wT'}\right] \\
&= F_H + \bar{\rho}\, c_p\, \alpha\overline{wT}^{12}
\end{aligned}$$

i.e. the error is of an order less than that of the terms neglected in deriving (7) and (8).

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The most serious instrumental error is probably that caused by the vane not following the most rapid changes in direction of the horizontal wind. This has two consequences. In the first place the angle of yaw may be great enough to carry air from the anemometer wires to the thermometer, giving spurious temperature fluctuations. Trials in the wind tunnel show that these fluctuations are large and have a characteristic appearance. We have detected such effects on a number of traces taken in the field in light winds; they do not occur in a large majority of the traces we have measured and we are confident that they do not noticeably affect our results. The second effect of yaw is to change the calibration of the anemometer in such a way that the indicated total wind increases, the indicated angle to the horizontal decreases rapidly, and the computed vertical component decreases, as the angle of yaw is increased. The effect has been investigated very approximately in the wind tunnel and Fig. 3 gives an indication of the magnitude of the error in the vertical component of wind for various angles of inclination and yaw. Fig. 4 allows an estimate of the effect of this on the computed flux. It shows the frequency of occurrence of various angles of yaw from the mean direction, determined on the site with a fixed horizontal $_{V}$ anemometer and 1/10 sec. galvanometers, in a mean wind of 450 cm sec.$^{-1}$ on 27th January, 1953, and the distribution of angle of inclination as indicated by the anemometer in normal use in a mean wind of 195 cm sec.$^{-1}$ during part of observation 11, Table I. The occasion used for distribution of yaw is not typical owing to the high wind speed, but is the only one available to us. A wider distribution would be expected with lighter winds, but of course the vane follows all but the most rapid excursions. The distributions of yaw and inclination are not independent but if we base an estimate of error on these curves assuming independence it should not be far wrong. It can be seen that the mean error in the vertical wind should be well below 10%, and we suggest this as a reasonable value for the degree to which the heat flux is underestimated due to all instrumental defects.

V. Results of other investigators

As we mentioned in Section I we know of only two other investigations in which the heat flux determined by the fluctuations method has been compared with a measurement or estimation obtained otherwise. By far the more extensive of these is the work carried out near Melbourne, Australia by a team from the C.S.I.R.O. Section of Meteorological Physics, preliminary reports of which have been published by Swinbank (1951, 1952). We have been informed in conversation that a very considerable amount of material has been collected and reduced, and that the conclusion drawn is that the fluctuations method yields a correct value for the heat flux - a result which we have been unable to obtain at Kew. The only results of which we have any detailed knowledge are however those published by Swinbank in his preliminary account; the relevant parts are reproduced here as Table IV, for the convenience of readers of this report.

Swinbank's method of test differs from ours in that he does not apply the Bowen ratio technique, but measures the flow of both heat and water by the fluctuations method. For those who maintain that the Bowen ratio method is unreliable in principle, Swinbank's test is the more satisfactory - it relies on the technique under test to give the sum of the energies used in evaporation and removed by convection, which sum is measured independently. It has however one weakness of principle - the expression for the flux of water analogous to (8) for heat is valid, but not those analogous to (7) and (9), as may be seen by substituting the water vapour concentration for temperature in the analysis of Section IV.2. The use of instrumental means in place of the elementary velocities and humidities when computing water flux is not analogous to the corresponding procedure for the heat flux and should strictly be justified by experiment e.g. by determining the water flux from means on different time scales. So far as his published work goes Swinbank does not seem to have done this.

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A glance at Table IV shows that only those measurements made with large incident solar radiation are of value in comparing Swinbank's results with ours. In his night observations the heat flux unaccounted for is, in all cases but one, much larger than the measured convective heat flux. His daytime observations check very satisfactorily, but it will be noted that the measured gradients of temperature and humidity are such that the Bowen ratio technique would give results for the heat flux differing by almost an order of magnitude from those computed from the fluctuations. If therefore we accept the latter fluxes as the correct values we see that Swinbank's measurements discredit the Bowen ratio method, and are in contradiction to the evidence presented in Section IV.1.

McCready's (1953) measurement of the heat flux was part of an extensive investigation aimed mainly at analysing the velocity fluctuations, and he reports only one run useful for our purpose. His apparatus responded to frequencies up to 50 c/s, but the length of record and details of analysis (except that this was automatic) are not clear from his paper. The observation was made over a desert surface, and the check value for the heat flux obtained from the net radiative flux and the ground conduction, i.e. evaporation could be neglected. He made simultaneous measurements at three heights with the result (approximate, from Fig. 10 of his paper)

$\rho c_p \overline{w'T'}$	at 15 cm	2.5×10^{-3}	cal cm^{-2} $sec.^{-1}$
$\rho c_p \overline{w'T'}$	at 70 cm	5.0×10^{-3}	cal cm^{-2} $sec.^{-1}$
$\rho c_p \overline{w'T'}$	at 240 cm	8.0×10^{-3}	cal cm^{-2} $sec.^{-1}$
check value		7.1×10^{-3}	cal cm^{-2} $sec.^{-1}$

McCready attributes this height variation to horizontal inhomogeneities at his site, but a single observation does not allow useful discussion of the point. It will be recalled that we have found no significant difference between measurements at 25 cm and 150 cm.

VI. Conclusions. Future work

The discrepancies between our two methods of determining heat flux have not been resolved, and the position is not clarified by appeal to the published results of other workers. Of the possible explanations which we discussed in M.R.P. 740 we have shown by the analysis of Section IV.2 and by the work on different time scales that the arguments concerning the periodicity of eddies mainly concerned in the heat transport were not well founded, and also that the results are not affected by changing the height at which observations are made. We have not settled satisfactorily whether the length of time for which the records are taken has a serious effect on the result.

We propose next to make observations on a site with much more uniform surroundings, and in conjunction with N.E. Rider to have simultaneous measurements of evaporation (by weighing) and of the drag of the surface on the wind. At the same time we will attempt to take longer unbroken series of observations, by modification of the recording mechanism. We propose to do this work with the 1/10 sec. and 2 sec. galvanometers.

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Appendix I

The apparatus used in conjunction with the short period galvanometers.

1. The properties of the galvanometers

The four galvanometers used for the high speed recording were a standard model made by Messrs. Pye. They have a nominal "response time" of 1/100 sec., a sensitivity of about 2 mm/μa at 1 metre, an internal resistance of about 50 Ω , and nominally require an external resistance of order 100 Ω for critical damping. It was proposed to operate them at optimum damping (i.e. 0.6-0.7 critical, giving a minimum of amplitude distortion and an approach to linear variation of phase distortion with frequency) so their characteristics were investigated fairly thoroughly. The free period was found to be close to 60 c/s in all cases, the internal resistance was as specified, but the behaviour with change of external resistance was rather different for the different galvanometers, and in one case the critical damping resistance was much less than that specified. The required damping resistance was found in each case by recording the response of the instrument with different parallel resistances to a step voltage. The determination was checked by measuring the response of the instrument with its appropriate damping resistance to a fixed current of varying frequency. Fig. 5 shows the result of one such test, the points being the response of the galvanometer relative to its D.C. response, the full curve being the calculated response for a free period of 60 c/s and damping 0.7 critical. (The wave form used in these tests differed considerably from a pure sine wave).

2. The recording of temperature

It seemed desirable to use the same sensing head and vane as in the previous work, but with the thermometer incorporated in this (4 cm of 0.0005 in. diam. Pt, resistance about 37 Ω) the sensitivity of the high speed galvanometers was not sufficient to give the required resolution on the temperature trace. An amplification of X5 or X10 was needed. A photo-electric method was used as shown in Fig. 6. Light from a 12v. 48 watt projector lamp passed through a metal grid and was focussed on the primary galvanometer mirror (which is about 2 mm x 1 mm) by an achromatic condenser. The divergent beam from the galvanometer mirror was intercepted by a second grid, a suitably enlarged copy of the first, which masked two selenium photocells arranged side by side in parallel opposition, the separation of the cells being variable. The output of the cells was applied to the secondary galvanometer, which gave the temperature record. Both galvanometers were correctly damped, and it was confirmed that the phase lag introduced by the amplifier system was that appropriate to two galvanometers.

The amplification was checked before and after each run by noting the deflections corresponding to variation of R 12, Fig. 6, in steps of 0.1 Ω. The amplification was found to be linear over a range of several degrees C, the central temperature of this range being variable by moving the masking grid and photocells. The degree of amplification was variable by controlling the D.C. voltage on the projector lamp, and so long as this voltage was held constant both gain and zero of the system were sufficiently stable over the half to one hour periods required.

We have not seen a record of the use of this type of amplifier (i.e. without electronic amplification) with high speed galvanometers, but the only difficulty not met with in more orthodox examples is introduced by the small area of the mirror. Our amplifier is mainly distinguished by its small gain, rudimentary optical system, and inelegant construction.

/The

The use of two damped galvanometers in the thermometer circuit introduces a phase lag between the wind and temperature traces; this is calculable from the circuit properties as 90° for a 60 c/s input. The value was checked by recording a 50 c/s voltage simultaneously on a single galvanometer and through the amplifier. With the damping chosen to give linear phase distortion this means that the temperature trace lagged 1/240 sec. behind the wind traces. As readings were taken at 1/30 sec. intervals only, and in view of the measurements on the time constant of the sensing wires (see below), this lag was ignored.

3. The time constant of the sensing wires

It is assumed when computing the heat fluxes that the nature of the traces obtained is controlled entirely by the time constants of the galvanometers and their associated circuit, the effect of the sensing wires being negligible. This was clearly true for the 1/10 sec. and slower galvanometers, but it was thought necessary to investigate the response speed of the wires before using the 60 c/s galvanometers. The device used was that described by Grant (1950). The wire under investigation is used as a resistance thermometer in one arm of an audio frequency bridge, the out of balance voltage being recorded on a C.R.O. A heating current is passed through the wire for a time with the rest of the bridge out of circuit. The heating current is switched off, the bridge switched in, and the decay of the out of balance voltage recorded. This process is repeated cyclically, the C.R.O. being synchronised to it. The only difference between our apparatus and Grant's was that we used an available 1000 c/s fork-controlled oscillator as the bridge voltage source, in place of his 2000 c/s oscillator. Tests were made in the wind tunnel at various wind speeds. Examples of the records obtained are shown in Fig. 7. The variation of lag, i.e. the time for recording of a fraction (1 - 1/e) of a step function disturbance, with ventilation speed is shown in Fig. 8, from which we conclude that the use of the 60 c/s galvanometers is justified in wind speeds of about 100 cm sec.$^{-1}$ and upwards. It is clear that the galvanometer period cannot usefully be further reduced at these wind speeds.

4. Apparatus used to determine the heat flux by the Bowen ratio method

Certain changes were made in this apparatus in connection with observation 13, Table I, but no change of principle was involved. In place of the mast of 7 wet and dry bulb thermocouples previously used to determine the temperature and humidity profiles two electric Assmann psychrometers were used, at heights of about 110 and 160 cm. They were read by eye at about 1 min. intervals. The net radiative flux was measured by a carefully calibrated aspirated plate recording radiometer, of the type described by Dunkle, Shimazaki, Gier and Possner (1949). This allowed the work to be done in long sunny intervals which occur on days with appreciable cumulus cloud cover (up to about 4/10), when the atmospheric radiation cannot be determined by a radiometer of limited aperture, such as the Linke-Feussner previously used.

Appendix II

Determination of the heat flux from the whole and parts of a series of consecutive observations.

In Fig. 2 F_n/F_p is plotted against n/p for three sets of data obtained in very different circumstances (F_n is the mean value of the heat flux determined by applying formula (9) to sets of n consecutive observations in a series of p observations). The fact that these very different experiments should give the same curve was so unexpected that we have examined the procedure to see if the cause is algebraic rather than physical. This does not seem to be the case; the curve is related to certain other similarities which we have noted in observations made on different time scales. The first of these, discussed in this paper, is that the observations made with the 1/60 sec., 1/10 sec., and 2 sec. galvanometers give the same result for the heat flux; the second, for which we hope to produce evidence in a later report, is that the serial correlation functions of sets of observations of vertical velocity taken on these three different scales are similar.

The following crude analysis of the form of F_n/F_p applies when the individual observations in the series of p are used once only in computing F_n, i.e. when the separate series of n observations do not overlap. In Fig. 2 this is true of the points for which $n/p < 0.25$. Dropping for convenience the constant multiplier $\bar{\rho} c_p$ we write

$$F_p = \frac{1}{p} \sum_{p} w'T'$$

and

$$F_n = \frac{1}{p} \sum_{a=0}^{r-1} \left[\sum_{an+1}^{n(a+1)} w'T' - \frac{1}{n} \sum_{an+1}^{n(a+1)} w' \sum_{an+1}^{n(a+1)} T' \right] \quad (p = n-r)$$

$$= F_p - \frac{1}{pn} \sum_{a=0}^{r-1} \left[\sum_{an+1}^{n(a+1)} w' \sum_{an+1}^{n(a+1)} T' \right]$$

$$\equiv F_p - \frac{1}{rn^2} \sum_{a=0}^{r-1} \alpha_a \beta_a \equiv F_p - \frac{1}{n^2} \overline{\alpha_a \beta_a}$$

$$\equiv F_p - F_r^*$$

F_r^* is the heat flux computed from r sets of simultaneous means over n readings. We note in passing that Fig. 2 shows that in general $F \neq F^*$, but the difference between them is not large.

Similarly we have

$$F_{2n} = F_p - \frac{1}{p.2n} \sum_{a=0}^{r/2-1} \left[(\alpha_a + \alpha_{a+1})(\beta_a + \beta_{a+1}) \right]$$

$$= F_p - \frac{1}{2^3 n^2} (2 \overline{\alpha_a \beta_a} + 2 \overline{\alpha_a \beta_{a+1}})$$

/assuming

assuming the series of α and β to be homogeneous, and generally

$$F_{qn} = F_p - \frac{1}{q^2 n^2}\left(q\,\overline{\alpha_a \beta_a} + 2(q-1)\,\overline{\alpha_a \beta_{a+1}} + 2(q-2)\,\overline{\alpha_a \beta_{a+2}} + \text{-----} + 2\,\overline{\alpha_a \beta_{a+q-1}}\right).$$

We now write

$$A_1\,\overline{\alpha_a \beta_a} = \overline{\alpha_a \beta_{a+1}} \quad \text{etc.}$$

so that

$$F_{qn} = F_p - F_r^{*}\left(\frac{1}{q} + \frac{2(q-1)}{q^2}A_1 + \frac{2(q-2)}{q^2}A_2 + \text{-----} + \text{-----}\right)$$

or

$$\frac{F_{qn}}{F_p} = 1 - \frac{(F_p - F_n)}{F_p}\left(\frac{1}{q} + \frac{2(q-1)}{q^2}A_1 + \frac{2(q-2)}{q^2}A_2 + \text{-----} + \text{-----}\right).$$

We see from this that two series of simultaneous values of w' and T' will give the same curve in Fig. 2 if F_n/F_p is the same for one value of n/p, and the coefficients A are the same. For each series these coefficients depend on the correlations between an observation or mean of vertical wind and the simultaneous and subsequent observations or means of temperature. If these correlations are the same whatever the time scale of observation (or number of observations involved in the mean) the results shown in Fig. 2 follow. We have not computed any such correlations; we do however find that the correlations of vertical wind and simultaneous temperature $\overline{\alpha_a \beta_a}/\sigma_\alpha \sigma_\beta$ and the serial correlation function for vertical wind $\overline{\alpha_a \alpha_{a+x}}/\overline{\alpha^2}$ are similar on the three scales of observation we have used, and it seems reasonable to accept this explanation of the facts displayed in Fig. 2. If we do this it seems that we cannot use arguments based on Fig. 2 either to draw conclusions concerning the length of the series of observations necessary to give a satisfactory value for the heat flux, or to indicate the extent to which the observations we have made underestimate the true heat flux.

/TABLE I

TABLE I

Results of the heat flux computation

Observation No.	Date	Type	Height cm	Galvanometer period sec.	Duration of record sec.	Readings per record	No. of records	Heat flux (1) $\bar{\rho}\, c_p \overline{w'T'}$ mw cm^{-2}	Heat flux (2) by Bowen ratio mw cm^{-2}	Ratio (1)/(2)
1	25 Oct. 50	Inversion	25	0.1	40	200	7	-4.2±1 *	-9.2	0.46
2	11 Dec. 50	Inversion	25	0.1	40	200	5	-1.7±0.3	-5.5	0.31
3	24 Apr. 51	Isothermal	25	0.1	50	200	2	-0.2±1	(0)	-
4	2 June 51	Lapse	25	0.1	50	200	7	+7.1±2	+21	0.34
5	6 June 51	Lapse	25	0.1	50	200	1	+7.1	+13.5	0.53
6	9 June 51	Lapse	25	0.1	50	200	1	+7.1	+12.5	0.57
7	2 July 51	Lapse	25	2	240	48	3	+2.5±0.6	+8	0.32
8	23 Oct. 51	Inversion	25	0.1	50	200	6	-1.7±0.3	-5	0.34
9	30 Apr. 52	Lapse	140	0.1	50	200	7	+4.6±1	+8	0.58
10	22 July 52	Lapse	140	0.1	50	200	5	+12.6±1	+20	0.63
11	28 Aug. 52	Lapse	140	0.1	50	200	5	+3.0±1.5	+6	0.50
12	11 Nov. 52	Inversion	140	0.1	50	200	6	-1.3±0.5	-4.5	0.29
13	9 June 53	Lapse	140	0.017	10	300	5	+6.9±1.8	+11.5	0.60

* The mean deviation of the heat fluxes calculated from the individual records is given.

TABLE II

Relation between the eddy conductivity for water (K_w) heat (K_H) and momentum (K_M) (after N.E. Rider)

Rider's Observation No.	K_w / K_M	K_H / K_M	K_H / K_w
29		1.40	
30		1.49	
31	1.07	1.13	1.06
32	1.09		
33	1.34	1.63	1.22
34	1.34		
35			2.80
37	1.23		
38	1.47		
39	1.02		
40	0.83		
43	1.25		
44	1.67	1.96	1.18
45		1.10	
46	1.23	1.35	1.09
48		2.10	
50		1.28	
Mean	1.23	1.48	1.14
Mean Deviation	±0.17	±0.27	±0.06
Estimated S.Ds.	±20%	±26%	±23%

TABLE III

Values of heat flux from complete (F_p) and partial (F_n) series of observations (factor $\overline{\rho}\, c_p$ omitted)

Date	p	Chart No.	F_p	F_n for n = 5	25	50	75	100	150	200
22 July 52	200	2	10.6		4.9	8.0	8.6	8.2	7.8	
		6	9.2		3.2	6.3	8.8	8.0	8.5	
		7	9.8		8.5	9.3	9.9	9.5	9.7	
		8	8.4		4.2	7.5	9.6	9.1	10.3	
		9	9.3		4.9	6.6	8.3	7.1	7.1	
		Mean	9.5		5.1	7.5	9.0	8.0	8.7	
9 June 53	300	2	3.1	0.3	1.5	1.8		2.2	3.4	4.1
		3	7.0	1.6	5.1	6.8		7.6	7.7	6.6
		4	6.0	0.2	0.4	2.0		3.4	3.8	5.2
		5	8.6	0.7	5.7	7.1		5.9	8.8	7.7
		7	3.9	0.3	0.9	0.9		3.3	3.2	3.3
		Mean	5.7	0.6	2.7	3.7		4.4	5.4	5.3

TABLE IV

Swinbank's measured values of heat, water vapour, momentum and radiative heat fluxes, and calculated transfer coefficients. Abridged from W.C. Swinbank (1952)

Date and Time (BSB)	$(\frac{\partial T}{\partial z} + \Gamma)$ (°C/cm x 10^4)	Humidity gradient (gm/gm/cm x 10^6)	Gradient of momentum per unit mass per second x10^2	Fluxes of (all positive upwards): Sensible heat (cal/cm^2/sec. x 10^4)	Water vapour (gm/cm^2/sec. x 10^7)	Latent heat (cal/cm^2/sec. x 10^4)	Momentum (gm/cm/sec.^{2})	Radiative flux (cal/cm^2/sec. x 10^4): Net short wave	Long wave from ground	Long wave from atmosphere	Heat flux into ground (cal/cm^2/sec. x 10^4)	Warming of layer of grass (cal/cm^2/sec. x10^4) estimated.	Vertical flux of heat unaccounted (cal/cm^2/sec. x 10^4)	Transfer coefficient (cm^2/sec./x 10^{-2}): K_H	K_W	K_M
5.3.51 1145	-34.3	-10.9	131	+23.4	+82.0	49	-1.94	118	122.4	85	14	3	+5	22.3	6.4	11.6
5.3.51 1200	-38	-19	123	+15.0	+68.9	40.3	-1.19	129	122	86	24	5	8	12.9	2.83	7.5
7.3.51 2035	28.6	-1.23	42	-3.2	-0.26	-0.16	-0.25	-	89.1	70.2	-18	-4	6	3.7	-	4.7
8.3.51 2050	25.0	1.02	-	-1.0	-1.3	0.7	-	-	89.5	72.6	-8.4	3	4	1.35	0.97	-
8.3.51 2135	47.0	1.69	-	-0.9	-0.8	-0.5	-	-	88.6	73.5	-18	-3	7	0.69	0.37	-
14.3.51 2125	110	-1.64	52	-1.2	1.1	0.6	-	-	98	93.0	3.0	1	0.5	0.35	0.52	-
15.3.51 1130	-53	-7.5	59	32	48	29	-0.53	166	131.3	89.7	47	10	6	19.3	5.1	7.0
15.3.51 1240	-71	-	71	28.6	-	-	-0.63	135	129.0	89.1	47	10	-	13.0	-	7.0
15.3.51 2115	17.5	-1.06	-	0.18	1.7	1.0	-	-	-	-	-	-	-	-	-	-

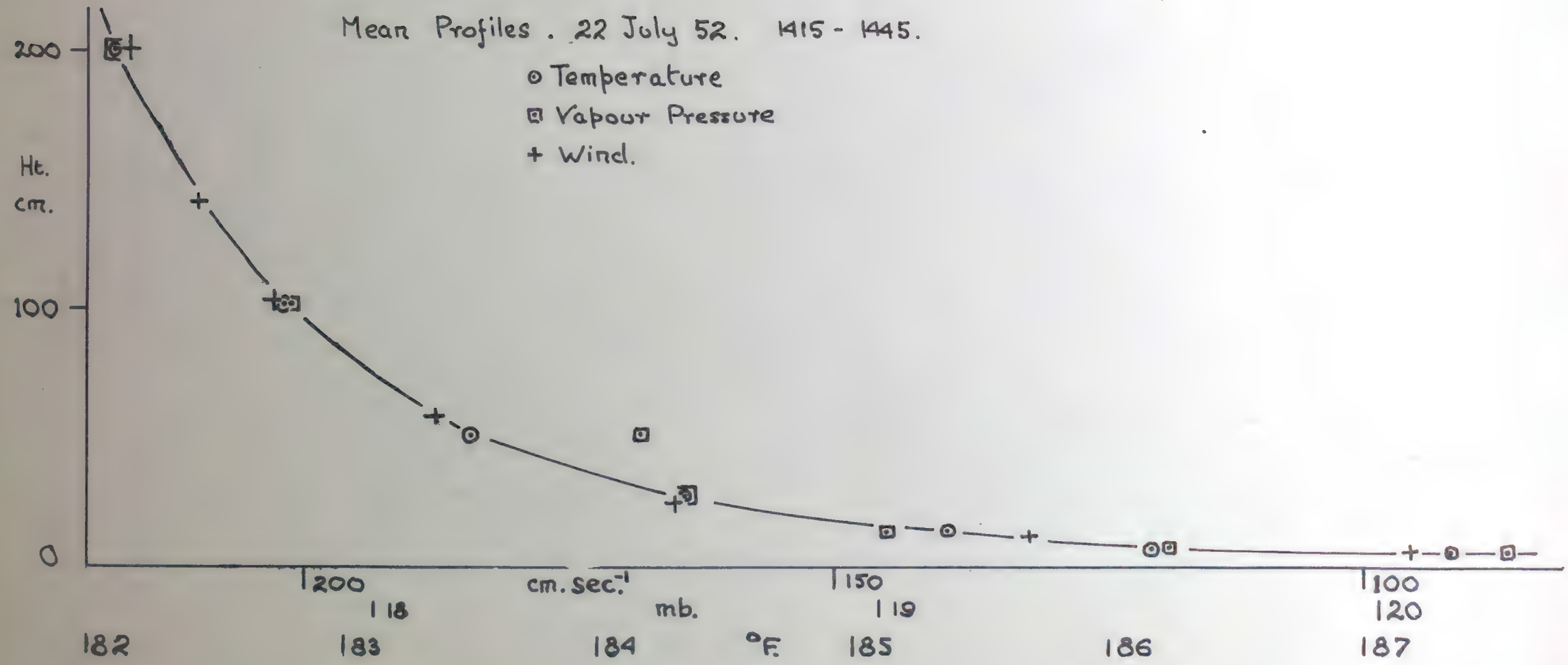
Fig. 1.

Mean Profiles . 22 July 52. 1415 - 1445.

⊙ Temperature

⊡ Vapour Pressure

\+ Wind.



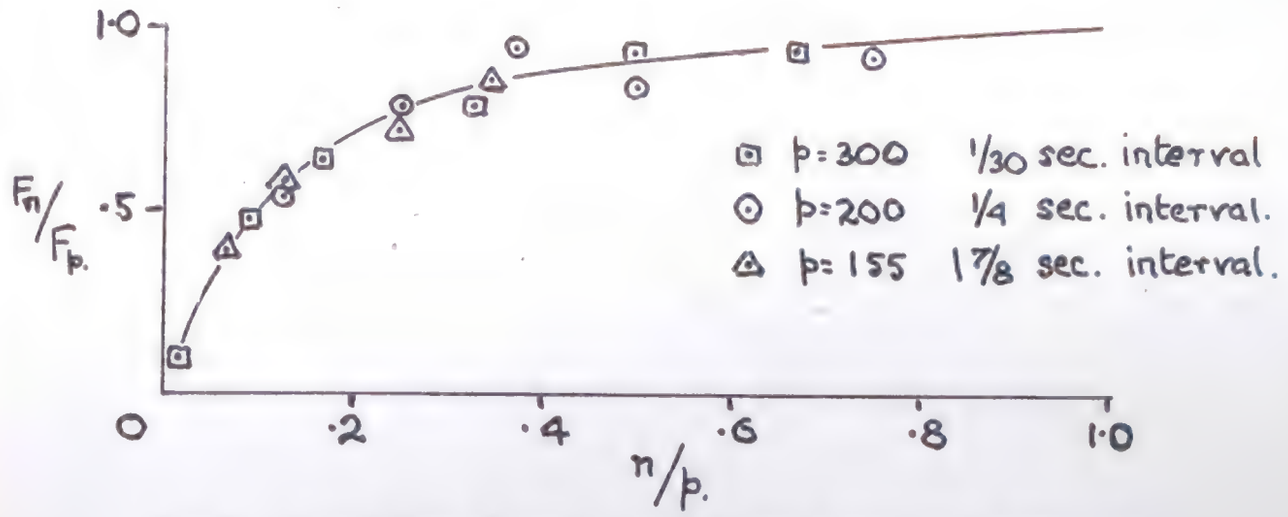


Fig.2. Heat flux from partial and complete series

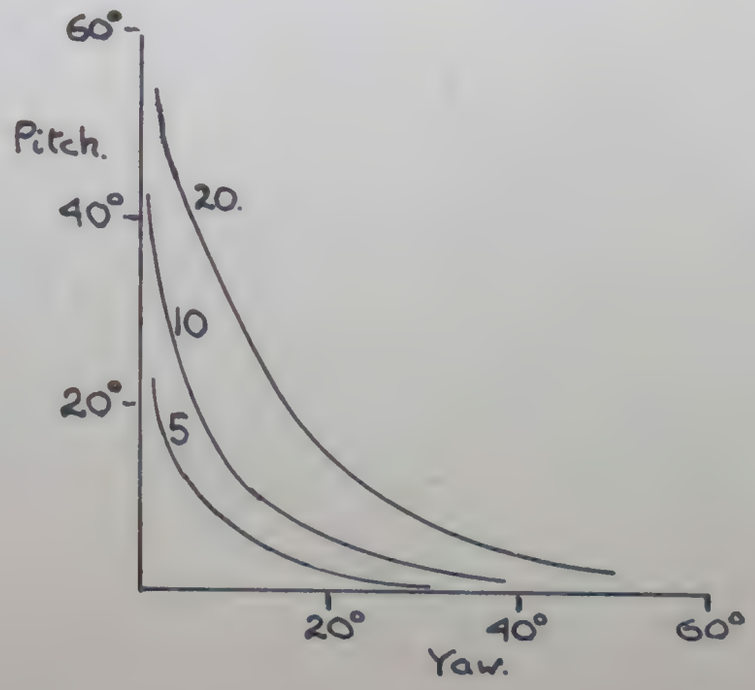
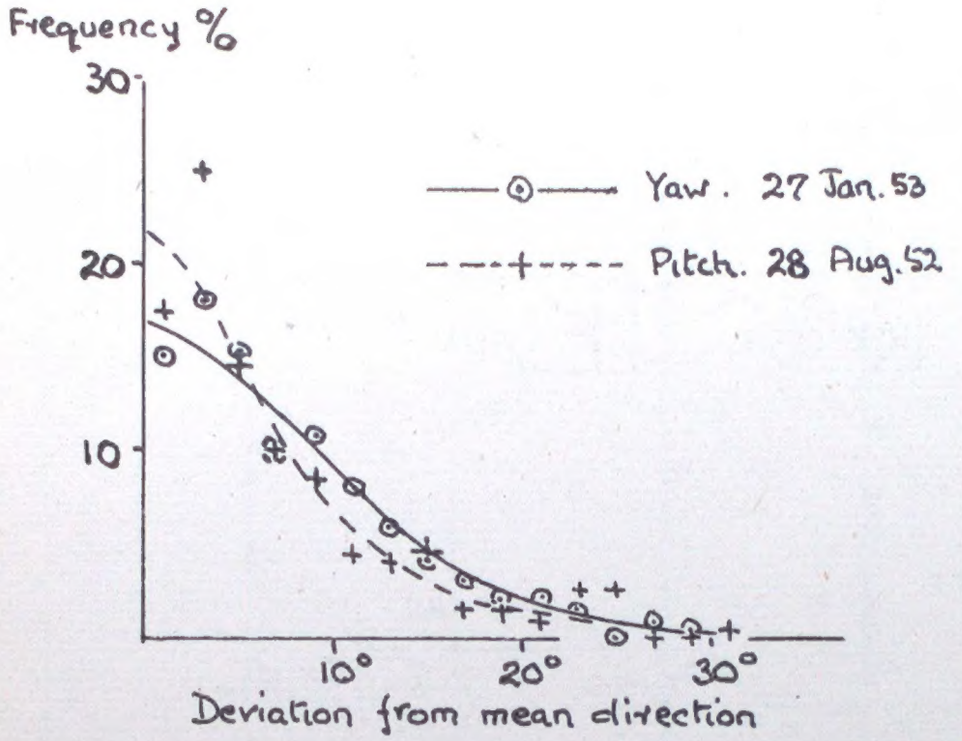


Fig.3 Approximate isopleths of error (per cent) in vertical component of wind at various angles of attack

Fig.4. Examples of frequency of deviations from mean wind direction in 1 min run



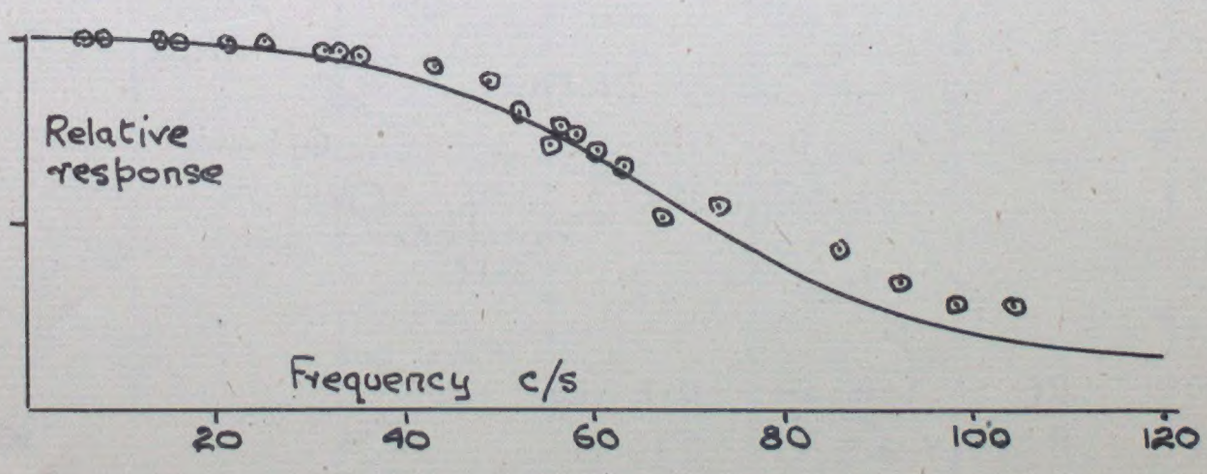
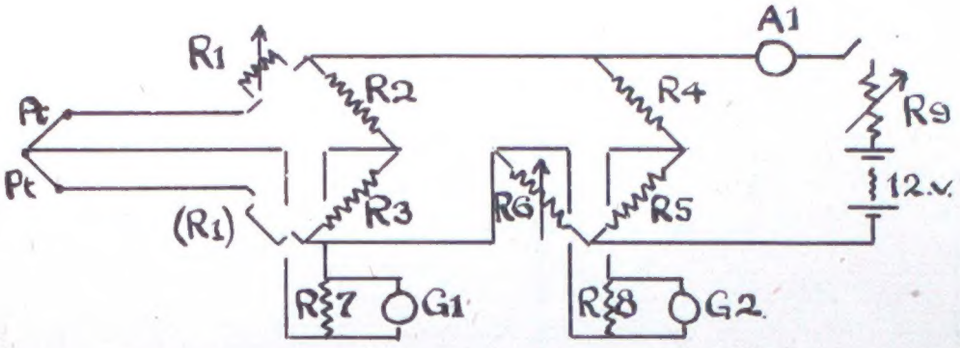


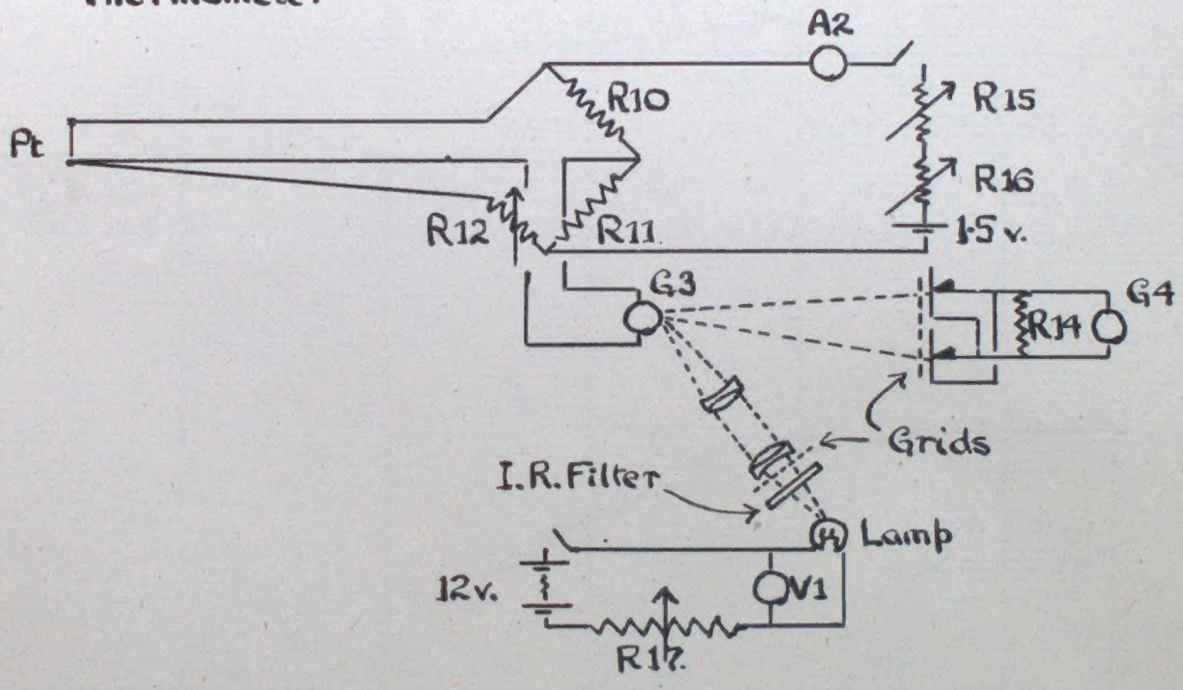
Fig.5. Response of damped galvanometer (Pye 19997). Damping resistance 135 Ω. (Full curve is theoretical response at 0·7 critical damping)

Fig.6. Circuit for high speed galvanometers

Anemometer



Thermometer

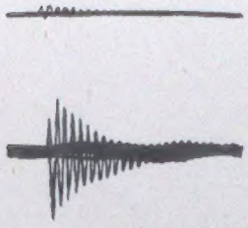


R1	Slide wire	0-1	ohm.			
R2,3,4,5.		500	"	R14		165 "
R6	Decade	0-111	"	R15		5000 "
	Slide wire	0-0·1	"	R16		50 "
R7		180	"	R17		0-1 "
R8		155	"			
R9		0-115	"	A1	Ammeter	0-200 mA.
R10,11		50	"	A2	"	0-5 mA.
R12	Decade	0-111	"	V1	Voltmeter	0-15 v.

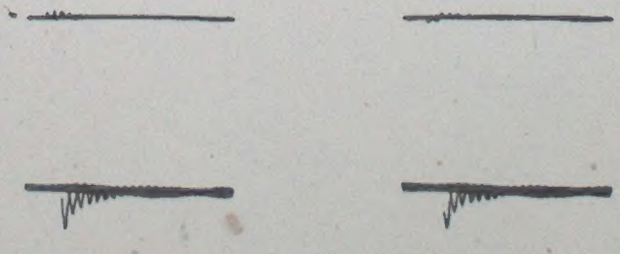
Galvanometers

G1	Inclination of wind	50·9	ohm.
G2	Wind speed	48·0	"
G3	Temperature primary	48·7	"
G4	" secondary	51·3	"

$u = 40\ cm.sec^{-1}$



$u = 97\ cm.sec^{-1}$



$u = 250\ cm.sec^{-1}$

Fig. 7

Determination of time constant of 0·0005 in. diam. Pt. wire

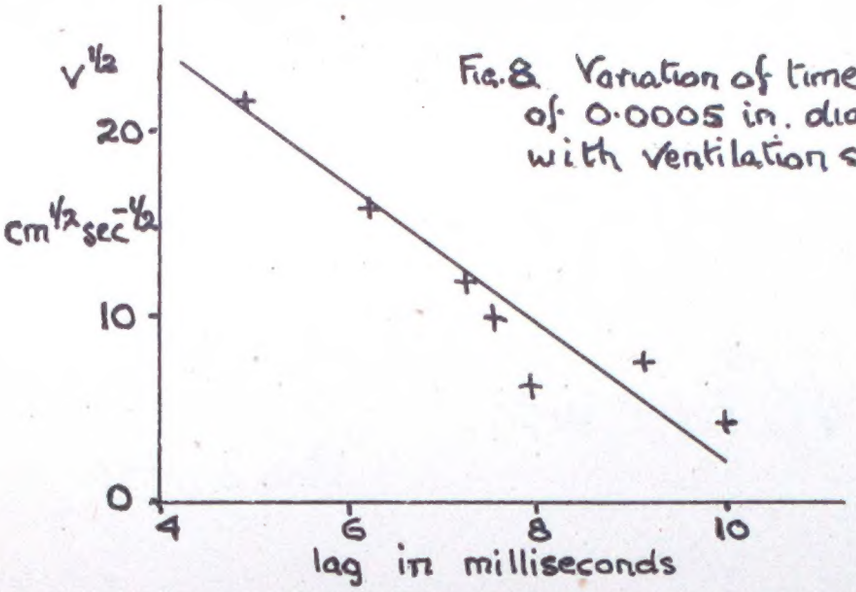


Fig. 8 Variation of time constant of 0·0005 in. diam. Pt. with ventilation speed